Repeated auctions for robust task execution by a robot team[☆]

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Abstract

We present empirical results of an auction-based algorithm for dynamic allocation of tasks to robots. The results have been obtained both in simulation and using real robots. A distinctive feature of our algorithm is its robustness to uncertainties and to robot malfunctions that happen during task execution, when unexpected obstacles, loss of communication, and other delays may prevent a robot from completing its allocated tasks. Therefore tasks not yet achieved are resubmitted for bids every time a task has been completed. This provides an opportunity to improve the allocation of the remaining tasks, enabling the robots to recover from failures and reducing the overall time for task completion.

Key words: Task allocation, auctions, multi-robot teams

1. Introduction

An autonomous team of robots may be deployed in a situation that is dangerous or inaccessible to humans, such as a building collapsed during an earthquake. The robot team can be used to map the building, identify unsafe areas, and locate and rescue survivors. Robots in the team will have different tasks. The tasks could be assigned to each robot before deployment, but this would reduce the team's ability to adapt to the situation. Thus it is preferable to have the robots determine the task assignments dynamically through negotiations within the team.

In this paper we propose a method for distributing tasks dynamically among a group of cooperating robots. We are interested in situations where each task can be done by a single robot, but sharing tasks will reduce the time to complete the tasks and thus has the potential to increase the efficiency of the robot team.

What makes task allocation to robots challenging is the fact that robots have to physically move to reach the locations of their assigned tasks, hence the cost of accomplishing a task depends not only on the location of the task itself but also on the current location of the robot.

In this paper we present empirical results obtained both in simulation and with real robots using the algorithm we originally presented in [17]. The algorithm, which is based on auctions, does not guarantee an optimal allocation, but is specially suited to dynamic environments, where execution time might deviate significantly from estimates, and where the ability to adapt dynamically to changing conditions is crucial.

The algorithm is fully distributed. There is no central controller and no central auctioneer, each robot auctions its own tasks and clears its own auctions. The only assumption we make is that the robots can communicate with each other.

The auction mechanism we propose is based on a combination of *sequential single-item auctions* [16, 14] and *repeated parallel single-item auctions* [6]. It attempts to minimize the total time spent to complete the tasks by minimizing the sum of the path traversal times for all the robots and by imposing a time limit for task execution. With the simplifying assumption of constant and equal speed of travel for all the robots, this is equivalent to minimizing the sum of path lengths over all the robots, as the MiniSUM objective in [19]. The time limit can force reallocation of tasks, hence the algorithm's secondary objective is to minimize task completion time (called MiniMAX objective in [19]).

The algorithm we present is simple but robust to failures during execution. It aims at finding a compromise between computational complexity, quality of allocations, and ability to adapt. If a robot finds an unexpected obstacle, experiences any other delay, loses communication, or is otherwise disabled, the rest of the team continues to operate.

In this paper we describe the algorithm, analyze its complexity, and report empirical results obtained both in simulation and with real robots in a variety of environments.

2. Related Work

A recent survey [8] covers in detail the state of the art in using auctions to coordinate robots for tasks such as exploration [13], navigation to different locations [19], and box pushing [10]. Auction-based methods for allocation of tasks are becoming popular in robotics [11, 19] as an alternative to other allocation

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methods, such as centralized scheduling [3], blackboard systems [9], or application-specific methods, which do not easily generalize [1] to other domains.

Combinatorial auctions, where combinations of tasks are bid at once, have been used to allocate navigation tasks to robots [2]. The resulting allocation is optimal but due to the computational complexity of combinatorial auctions, generating bids and clearing them is slow, and they do not scale well.

Sequential single-item auctions [14, 16, 19] auction tasks individually, but robots bid on tasks accounting for their previous commitments. This type of auction can be computed in polynomial time producing solutions that, when the objective is to minimize the sum of the path costs for all the robots, are a constant factor away from the optimum [16]. Three objectives are examined: MiniSum, minimizing the sum of the path costs, MiniMax, minimizing the maximum path cost, and MiniAve, minimizing the average path cost over all the robots. The bidding rules are such that there is no need for a central controller. As long as each robot receives all the bids from all the robots, each robot can determine the winner of each auction. However, this requires each robot to keep track of its own costs and of the other robot costs, and so it is not robust to robot malfunctions. Robots are expected to know the exact cost of completing each task at the start. It is unclear how changes to this cost caused by unexpected changes during execution can be handled.

Repeated parallel single-item auctions [6] auction each task separately and treat it as independent of other tasks. The auctions are repeated periodically after a fixed time interval. These auctions are fast to compute and more robust. They make use of a pulse that is sent out at fixed time intervals to all the robots to restart the single-item auction between robots. This enables robots to switch tasks if the allocation can be improved and helps in case of unexpected problems, but has the undesirable effect that the length of the entire path covered by the team might be unbounded [19].

We also use single-item auctions, and we repeat the auctions multiple times while the tasks are being executed. However, instead of repeating the auctions at regular intervals, we repeat them whenever a task has been completed. This reduces the need for communication and the time spent in clearing auctions, while still providing the ability to react to changes in the environment or in robot functioning, typically without degrading performance. In addition, we account for tasks won from an auctioneer before bidding on subsequent tasks from the same auctioneer, as in sequential single-item auctions. This reduces the chance of an oscillatory situation where tasks keep getting transferred back and forth between two robots, a problem that affects the total path length in repeated parallel single-item auctions. We discuss the complexity of our algorithm in Section 4.

Our approach is similar to the method presented in [7] where a group of robots is given a set of tasks and robots are selectively disabled in different manners in order to measure their performance, i.e. the percentage of tasks completed, under different conditions. Our approach differs in that we assume a time limit for task completion. Additionally we use robots that are simpler and more prone to errors, hence the ability to change task allocation is essential.

In Figure 1 we show an example of how a combinatorial auction, a sequential single-item auction, and a parallel single-item auction differ from each other. The figure shows how the methods would behave in an environment with 4 tasks, T_1, T_2 T_3 and T_4, and 2 robots, $R_1$and R_2. Since some auction methods are sensitive to task order, we assume in what follows that tasks are bid in the order T_1, T_2 T_3, and T_4.

The combinatorial auction (shown in Figure 1 with solid arrows) examines the bids for every possible combination of the tasks, and finds the optimal solution, which is to allocate task T_2 to R_1, and to send R_2 to do tasks T_4, T_3 and T_1, in that order. The second diagram shows how the sequential single-item auction (shown with dot-dash arrows) would work. After winning T_1, R_1 adds the cost of moving from T_1 to T_2, and this is more than the cost for R_2 to go to T_2. Hence, R_2 wins T_2. When T_3 and T_4 are auctioned, R_1 wins both as its cost of going to T_3 via T_1 and to T_4 via T_1 and T_3 is less than the cost of R_2 going from T_2 to T_3 and T_4. The ratio of path costs of the sequential single-item auction compared to the combinatorial auction is $1.079 : 1$. The parallel single-item auction (shown in the third diagram with dashed arrows) assigns T_1, T_3 and T_2 (assuming some path optimization is done) to R_1, while R_2 does only T_4. This is because R_1 starts closer to the three tasks, even though R_2 could accomplish T_3 and T_1 more easily, after completing T_4. The ratio of path costs in this case is $1.155 : 1$. Hence, the sequential single-item auction achieves a better solution than the parallel single-item auction, but it is sub-optimal compared to the combinatorial auction solution.

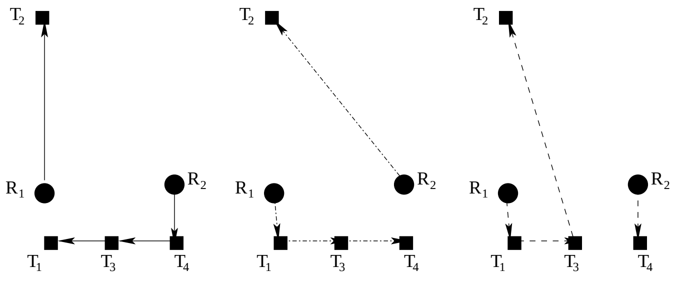

Figure 1: Task allocation using a combinatorial auction, a sequential single-item auction, and a parallel single-item auction

Our algorithm combines both the sequential single-item auction and repeated parallel single-item auction, by running several sequential auctions in parallel and repeating the auctions while the tasks are being executed in an attempt to improve task allocation. When tasks are initially put up for auction from a single source, the algorithm starts like the sequential single-item auction, but all remaining tasks are auctioned again after each task is completed. Therefore in the example in Figure 1 our algorithm will behave differently if an obstacle between two of the tasks is found after motion has begun; in that situation the tasks are re-allocated accounting for the modified path for that robot. If the tasks are initially distributed randomly between the robots, the allocation might be worse since the tasks of each robot are auctioned in parallel, but this will get corrected in the next auction after the first task (which is the nearest accessible task) is completed. Our algorithm is also geared to address failure on the part of one or more of the robots; the other robots will take over those tasks and finish them themselves.

3. Auction Algorithm

In this work we assume that each robot is given a map that shows its own location and the position of walls and rooms in the environment. No information is given about where the other robots are located and about other moving objects present in the environment, or about any temporary change, such as closed doors. The map is used by each robot to estimate, using Rapidly-exploring Random Trees [15], its cost of traveling to the task locations and to compute the path to reach them from its original location. Generation of RRTs is very fast, and scales well with large environments, so they are particularly appropriate for dynamic situations where computing the optimal path to achieve all the tasks allocated to a robot, as in [16], might not pay off, because tasks are likely to be reallocated. Examples of RRTs for our experimental setups are shown later in Figure 5 and Figure 6.

Each robot is also given a list of the robots in the team. We assume the robots can communicate with each other for the purpose of notifying potential bidders of auctioned tasks, submitting their own bids, and being notified when they won a bid.

Tasks are represented at a high-level by the location where the task is to be done and the cost of doing the task. Tasks are typically assigned by a user, but could be discovered autonomously by the robots themselves and added to the set of tasks as they are discovered. Tasks are assumed to be all equally important, but we have addressed elsewhere how to deal with tasks with priorities [18]. Robots typically do not know all the tasks, they are aware only of the ones assigned to them and discover the other tasks when they are auctioned.

Let R be the set of n robots $R = \{r_1, r_2, ...r_n\}$, and T the set of m tasks $T = \{t_1, t_2, ...t_m\}$, where each task is a location a robot has to visit. We partition the tasks into n disjoint subsets T_j, such that $\cup_{j=1}^{n} T_j = T$ and $T_i \cap T_j = \phi \ \ \forall i \neq j \ \ 1 \leq i, j \leq n$, and allocate each subset to a robot. Note that a subset can be empty.

The initial task distribution might not be optimal. Some robots might have no task at all while others might have too many tasks, the tasks assigned to a robot might be spread all over the environment, they might be closer to another robot, or may be unreachable by the robot.

A robot must complete all its tasks unless it can pass its commitments to other robots. To pass tasks to other robots, a robot puts its tasks into a Request for Quotes (RFQ) and broadcasts the RFQ to the other robots. A robot can choose not to bid on a particular task, based on its distance and accessibility to that task. Since the robots are cooperative and are trying to minimize task completion time, they will pass their commitments only if this reduces the estimated task completion time. The ability to pass tasks to other robots is specially useful when robots become disabled since it allows the group as a whole to increase the chances of completing all the tasks. Any task that cannot be completed by any of the robots, for instance because

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Repeat for each robot $r_i \in R$:
  1. Activate $r_i$ with a set of tasks $T_i$ and a list of the other
     robots $R_{-i} = R$ - $\{r_i\}$.
  2. Create an RRT using $r_i$'s start position as root.
  3. Find paths in the RRT to each task location in $T_i$.
  4. Assign cost estimate $c_j$ to each task $t_j \in T_i$ based on length
     of the path found starting from the current position.
  5. Order task list $T_i$ by ascending order of $c_j$.
  6. Establish communications with the other robots and build
     a list of all the tasks (system task list) for reference.
  7. $r_i$ does in parallel:
     (a) Auction its tasks:
           i. Create an RFQ with tasks in $T_i$.
          ii. Broadcast the RFQ to $R_{-i}$ and wait for bids for a
              fixed time limit.
         iii. Determine the lowest bid $b_{jk}$ among all the bids
              received for task $t_j$. Let $r_k$ be the robot that submitted
              the winning bid.
          iv. If $b_{jk} < c_j$ then send $t_j$ to robot $r_k$, else keep $t_j$.
              If $r_k$ does not acknowledge receipt, return $t_j$ to
              $r_i$. Mark $t_j$ as assigned.
           v. Ask $r_k$ to update its bids, if any, for the remaining
              tasks in $T_i$, ignoring tasks from other auctions
              ($r_k$ now has a new task). If $r_k$ does not
              acknowledge receipt, return $t_j$ to $r_i$.
          vi. Repeat from Step 7(a)iii until all tasks are assigned.
     (b) Bid on RFQs received from other robots:
           i. Find a RRT path for each task $t_r$ in the RFQ.
          ii. Compute cost estimate $c_r$ for each $t_r$ to which
              the robot found a path, starting from its current
              position.
         iii. If a bid is won, recompute the bids for the remaining
              tasks in that RFQ, accounting for the
              tasks assigned from that RFQ and submit bids
              to the auctioning robot (This ignores tasks from
              other auctions happening in parallel).
     (c) Begin execution of the assigned tasks:
           i. Find a path in the RRT to the first task ($t_j$) and
              start following it as closely as possible.
          ii. If new tasks are added as a result of winning new
              auctions, insert them in $T_i$ keeping $T_i$ sorted in
              expected execution order, from the nearest task
              to farther away ones, and repeat from Step 7(c)i.
         iii. If stuck or unable to complete the current task
              within the time promised in the bid plus a grace
              period, start a new auction to reassign its tasks.
          iv. If $t_j$ is completed successfully, notify all robots
              of task completion, update the system task list,
              and restart from Step 4.
until timeout or all tasks are completed.
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Figure 2: Task allocation algorithm.

it is not accessible, is abandoned. We assume that there is value in accomplishing the remaining tasks even if not all of them can be completed.

This process is accomplished via multiple single-item reverse auctions, in which the lowest bid wins. Auctions are run independently by each robot for its own tasks. The algorithm that each robot follows is outlined in Figure 2.

Each bid submitted by a robot is an estimate of the time it would take for that robot to reach that task location (assuming for simplicity a constant speed) from its current location.

Auctions are parallel, i.e. many auctioneers put up their auctions at the same time, but since a bidder generates bids in each auction independently of the other auctions, the effect is the same as having each auction done as a single-item auction that the bidder either wins or loses. Since a robot can bid for tasks in multiple parallel auctions, the order in which tasks are executed might be different from the order in which bids for tasks are submitted and won. The robot cannot compute its bids according to the order of execution, since the order is unknown at the time of bidding. Therefore, the robot treats each auction in a round in isolation. It computes its bids for each parallel auction assuming it starts at its current location, taking into account tasks that were won in that auction, but ignoring tasks won in other auctions. This can result in bids that over- (or under-)estimate the true cost. However, because tasks can be reallocated in successive auctions, this does not impact the quality of the solution significantly.

In each auction the bid for the closest task is a correct estimate because the robot accounts for all the tasks it won in that auction. Bids for further away tasks might not be correct because of synergies or unaccounted costs among the tasks won in other parallel auctions. If a task closest to a robot's current task is incorrectly assigned to another robot, the robot would likely win that task back in the next round of auction (unless that task gets completed before the next round) since now the robot would have moved closer to that task.

Each bidder re-orders its tasks each time a new task is added to its set, and moves immediately towards the nearest task (i.e. the task with the lowest cost) in its current entire set of tasks. Since auctions from different robots are done in parallel, the nearest task could be awarded after the robot started moving. In this case when the robot reorders its tasks it would discover it has now a nearer task and therefore change its current destination.

When the robot completes its current task, it starts a new auction for its remaining tasks. In addition to improving task allocation this is specially useful when a robot gets delayed, because this redistribution of tasks enables it to change its commitments and to adapt more rapidly.

The robots are given a time limit to complete each task, so that they do not keep trying indefinitely. Typically we use a grace period of 10 seconds over the time used in the bid for that task. If the task is not completed in that time limit, the robots start a new auction to allow a change in allocation of that task. When all the achievable tasks (determined by whether at least one robot was able to find a path to that task) are completed, the robots idle until the time given to them is over.

4. Auction Analysis

In analyzing the auction algorithm described in the previous Section we make the following assumptions: (1) all robots are working, (2) communications is perfect, (3) all tasks are accessible, and (4) all tasks are initially assigned to a single robot.

Formally, the problem is defined as follows: Given n robots and m tasks, the setup of the tasks can be represented as a graph G where tasks are the set of nodes T and paths between tasks are the set of undirected edges E. Each robot associates a cost with an edge. The cost measure we use is travel time. Since we assume constant and equal speed for all the robots, travel time is proportional to path length. As the auction algorithm proceeds, it assigns a subset of tasks T_j to each robot r_j, such that $T_j = \{t_j | t_j$ is assigned to r_j and $t_j \in T\}$ and all tasks are assigned, i.e. $\cup_{j=1}^{n} T_j = T$.

Each robot r_j needs to find a path to the task subset T_j assigned to it. This is equivalent to solving the traveling salesman problem for that robot. An approximation can be made using a greedy path algorithm that takes the shortest path to the nearest unvisited node. This has provable bounds, as follows. Build a Minimum Spanning Tree (MST) over T_j rooted at the node nearest to r_j. Let the sum of costs of edges in the MST be denoted by K_j. Then, the greedy path algorithm has a cost bound of $2 \times K_j + C_j$ where C_j is the cost for the robot to reach the root of the MST (from [5], Chapter 35, Sect. 35.2.1).

The overall team cost is then bounded by

$$C_{total} = \sum_{i=1}^{m}(Ct_i) + \sum_{j=1}^{n}(2 \times K_j + C_j)$$

where Ct_i is the individual task cost for task t_i. For simplicity we assume task costs $Ct_i = 0$ for $1 \leq i \leq m$.

The objective is to find an allocation S over T, such that S minimizes $(2 \times K_j + C_j)$ for $1 \leq j \leq n$, subject to the constraint $time_{total} \leq timelimit$. If multiple solutions are found with the same minimum cost, the solution which minimizes $time_{total}$ is chosen.

In Table 1 we compare the computational complexity of our algorithm with the complexity of sequential single-item auctions and repeated parallel single-item auctions. d is the sum of the path costs for all the robots in the optimal solution, i.e., the one that minimizes the sum of path costs for all the robots. i is the communication pulse interval, i.e. a signal broadcast to all the robots which triggers a new round of auctions [7]), and t is the completion time to execute all the tasks. Since the initial task allocation in our algorithm matches that of a sequential single-item auction [16], we can use their complexity analysis results to our algorithm. Subsequent auctions can result in added path costs; these are accounted for in our complexity analysis, as shown next.

4.1. Analysis of Path Length

In our algorithm, items are sold individually and each robot accounts for tasks it already won from the current auctioneer before bidding further on new tasks from the same auctioner. To bid on a new task a robot computes the difference in the cost

Method	Time complexity	Sum of Path Costs	Initial Comm.	Overall Comm.
Sequential Single Item Auctions [16]	$O(n \times m)$	$2 \times n \times d$	$n \times m$	N/A
Repeated parallel single item auction [7]	$O(n \times m)$	unbounded	$n \times m^2$	$n \times m \times t/i$
Our Algorithm	$O(n \times m^2)$	$(3 \times n - 2) \times d$	$n \times m$	$n \times m$

Table 1: Performance comparison between auction methods. n the number of robots, m the number of tasks, d the total path cost for all the robots in the optimal solution, i is the communication pulse interval, and t the completion time to execute all the tasks.

of the path that includes the new task from its previously computed path cost, and bids that difference. This is similar to the method of bidding described in [16] for the MiniSUM objective (using the bidSumPath strategy, which bids based on an approximate shortest path through all the tasks to be completed by that robot), but differs in the handling of multiple auctions, where each auction is considered independently of the others. It also differs because the overarching objective is trying to complete all the tasks within the time limit, so if a task takes too long to complete, it may get reassigned to a different robot. Given a large number of tasks and few robots, the time limit might not be sufficient to complete all the tasks using the MiniSUM objective. Thus, the objective becomes similar to the MiniMAX objective in [16].

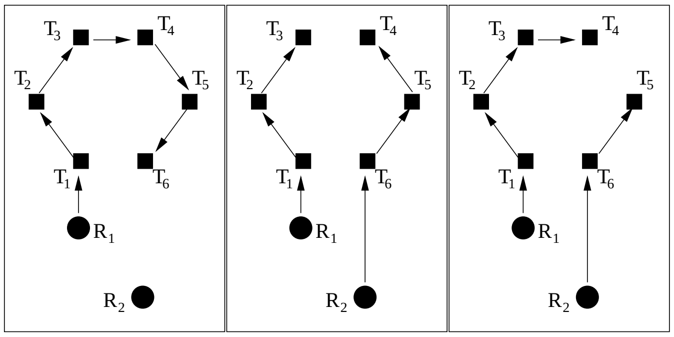


Figure 3: Task allocation for the MiniSUM objective (left), compared with MiniMAX (center) and with the allocation obtained by our algorithm (right).

Figure 3 shows a set of tasks to be completed by two robots. With MiniSUM, the first robot R_1 wins all the tasks. With MiniMAX, the two robots divide the tasks among themselves, to minimize the maximum path length traveled by each robot. Let's assume that the time to complete all the tasks using MiniSUM is longer than the time limit the robots are given. Then in our algorithm, after the initial MiniSUM allocation, robot R_1 would transfer some of its tasks to R_2. This results in the rightmost allocation, where R_1 has just enough time to complete tasks T_1 through T_4 and the remainder are taken by R_2.

The bound on the path cost after the initial minimization is equal to the MiniSUM bound of $2 \times d$ where d is the optimal path cost [16]. The reassignment of the tasks that exceeded the time limit would increase this bound as follows. Each of the tasks reassigned will go to a different robot, and can add a maximum of $2 \times d$ to that robot's path cost (since paths are recomputed to approximately minimize travel). Since the path length would decrease for at least one robot, the total increase in path length at most is $(n-1) \times 2 \times d$. Thus the bound on the path length is no more than the MiniMAX bound of $2 \times n \times d$ ([16]). When the number of tasks is large enough so that any allocation exceeds the time limit, our algorithm uses the MiniMAX objective. Therefore, the upper bound on the sum of path costs if the robots follow their initial allocation after the first auction is $2 \times n \times d$ ([16]). Following the initial allocation, in subsequent auctions, tasks may either stay with the same robot or be reassigned. With the exception of a special case (discussed next), reassignment is equivalent to having the initial auction with tasks in the reassigned order, and hence will only result in improvement. The special case occurs when two task allocations are nearly equivalent, and the robots keep switching between the two allocations in each auction. In this situation, since the number of auctions is limited by the number of remaining tasks, the maximum increase is $(n-2) \times d$ (the time taken by the remaining robots to reach those tasks). Thus, the bound on the cost becomes $(3 \times n - 2) \times d$.

Our auction method avoids the trap of parallel single-item auctions, where robots may all travel a long distance to reach a cluster of close together tasks, instead of having just one robot completing the tasks in that cluster [19]. This is achieved by making robots account for tasks already won from an auctioneer in any further bidding in the same auction. This ensures that if an auctioneer is auctioning tasks that are close to each other, the robot which wins one of those task from that auctioneer will continue to win subsequent nearby tasks from the same auctioneer. If nearby tasks were incorrectly given to a different robot previously, they will get reassigned to the closest robot, even when they are auctioned by different auctioneers. This is because independently of which robot auctions that task, the closest robot will bid its distance to that task, and will win the task since that would be the smallest bid. Over multiple auction rounds, this implies that tasks will tend to get assigned in groups to specific robots, based on their positions.

4.2. Analysis of Communications Complexity

The robots in our algorithm have more communication needs than the robots in [16], since in our case communication continues after the initial allocation, whenever there is an auction. There are n messages per auction, one per robot, and m auctions (The initial auction + 1 auction per task completed, with the exception of the last task). Therefore, a total of $n \times m$ messages are sent.

Failure of communication before the start of execution is a problem because tasks may never get shared between robots, and some tasks may remain undone. However, if communication failure takes place later, then the working robots can handle the additional tasks, and the problem can be treated as a modified one where the number of robots has gone down to $n-k$, if k robots are out of commission. Given k possible breakdowns, we need extra rounds of auctions for the tasks of the failed robots, thus resulting in $n \times m + n \times k = n \times (m + k)$ communication messages.

The number of messages we need is considerably smaller than the number needed for repeated parallel auctions, where tasks are placed for bidding continuously, so that communications takes place all the time.

5. Experimental Setup

We evaluated our algorithm through experiments done both in simulation and with real robots. Due to space and equipment constraints, we were limited to two robots for the real robot experiments, but were able to perform different and more complex experiments in simulation.

Experiments were performed with the Player/Stage [12] simulator. Player/Stage has the advantage that implementation details do not change significantly when shifting from simulation to real robots, thus making comparison easier. The experiments performed in our robotics lab used two Pioneer I robots, each mounted with a laptop and equipped with a wireless card for communication with each other. Communication was done through Java Sockets, since they provide features similar to what is available in the simulated system. The algorithm allows for dynamic addition of new tasks during execution, but for simplicity in the experiments the set of tasks and of robots is known at start and does not change during the execution.

Additional simulation experiments done earlier have been reported in [17, 18]. Their purpose was to evaluate the effectiveness of our auction algorithm in comparison to using a single initial auction, to measure the impact of loss of communication and of changes in the environment, and to measure the robustness of the algorithm. The earlier experiments used robots with 5 sonar sensors and differential drives, scattered in the hospital world environment provided by Player/Stage.

In this paper we report results on experiments conducted in three scenarios, a simplified building scenario we use for comparison of different algorithms in simulation, a lab scenario, where we performed experiments both with real robots and in simulation, and a more complex building scenario, where we performed experiments only in simulation.

5.1. Adaptations for Real Robots

There were some non trivial differences we had to deal with between the simulation and the real robot experiments.

1. Player 2.0 has significant differences in the way real robots move in comparison to the simulation. The same command produced in simulation a differing range of motion than when given to a real robot. Thus, motion commands had to be reconfigured to suit the robots.
2. Data for ranges of goals, sonar ranges, and collision ranges had to be modified to suit the real robots, since the form factor of the real robots was considerably different from that of the simulation.
3. In the simulation, all obstacles were detectable through sonars. In the real robot experiments, however, robots occasionally could not detect obstacles, such as table legs, because the sonar sensors were too far apart and missed the obstacle. This resulted in several collisions and near collisions in the real robot experiments, and produced far more variability in task completion times than what we had seen in the simulations. Details on the task completion times can be seen in Table 3 and Table 4.
4. Odometry in the real robots was significantly worse than that accounted for in the simulations. In most cases, unless there was a tight fit, the robots completed all the tasks without collision. Tasks were considered to be complete when the robots arrived within 30 cm of the task (i.e. an approximate robot-length away from the task). Collisions were tolerated in simulation; in the real runs, robots that had collided with obstacles were given one chance to recover and then shut down, to avoid damage.

6. Experimental Results

The main purpose of the experiments was to (1) compare the performance of our algorithm to other algorithms, specifically in terms of length of the solution paths found in different environments and under different conditions, such as presence of moving obstacles, (2) evaluate the performance of different aspects of our auction algorithm, such as auction time and communication overhead during execution, and (3) validate the simulation results by comparing simulation with real robots results.

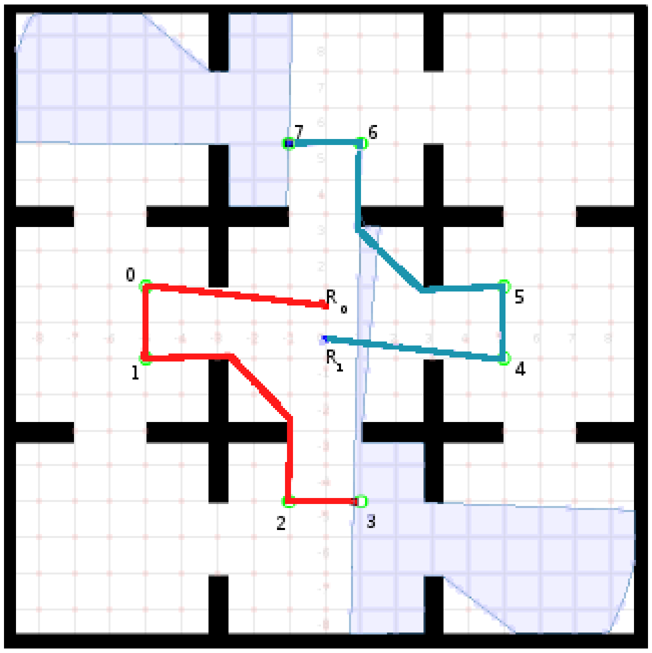
6.1. Simplified building scenario

We performed simulation experiments with a simple setup that would allow us to compare the following:

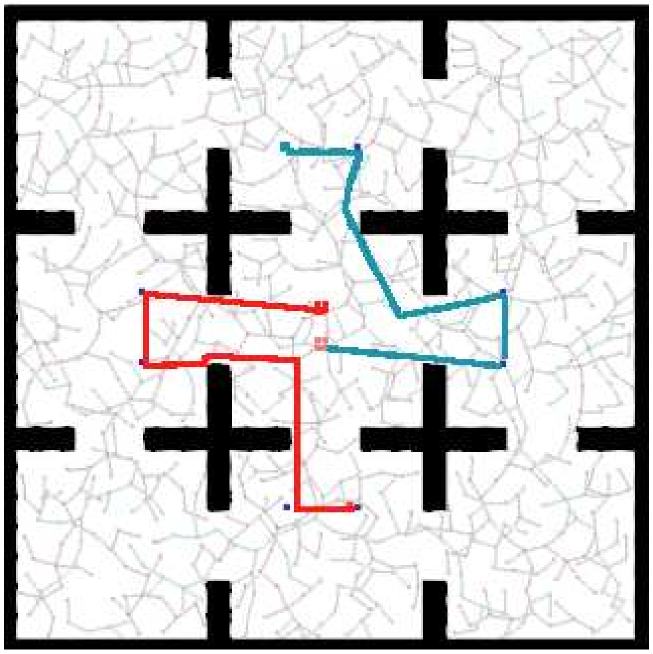
1. The optimal path length.
2. The path length given an initial assignment that is optimal, but using RRTs to compute the paths. RRTs do not produce the shortest path, so using them gives us a fair baseline for comparison. Since RRTs are generated for each run and they are different in each run they introduce additional variability in the path lengths across runs.
3. The path length using one round of parallel single item auction. Given the layout of the tasks, repeating parallel single item auctions would keep the same allocation of tasks to robots, so in this case a single round acts like repeated rounds.
4. The path length using our algorithm of repeated sequential single-item auctions.

Figure 4 illustrates the environment used, and the paths that were found by the different algorithms. The environment is 16×16 meters. We ran 20 experiments in each scenario.

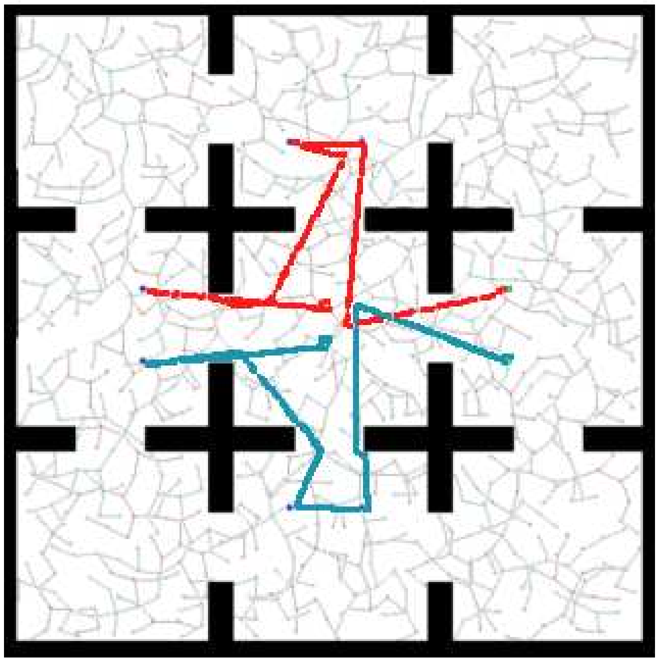
The results are summarized in Table 2. The results show the effect of using RRT-based path finding - while very effective in large environments with multiple rooms, in small rooms RRTs can produce suboptimal results. In this case, even if we start with a fixed optimal allocation, RRTs increase the average path



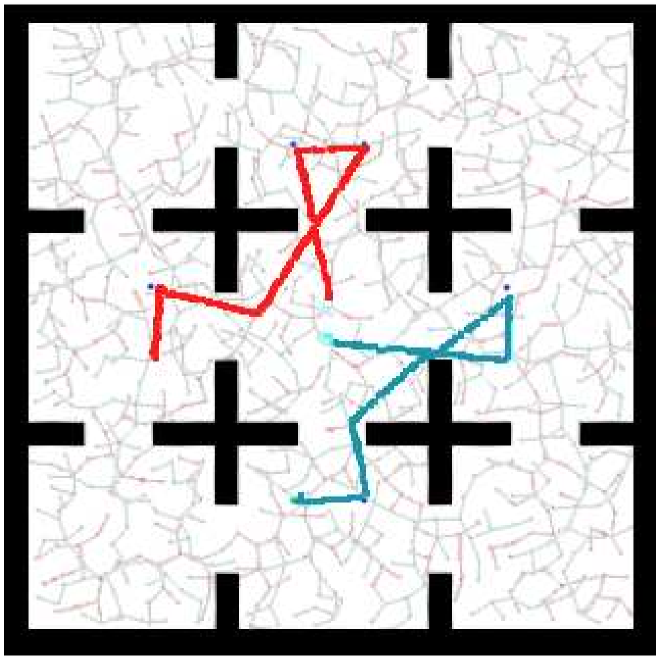
(a) Paths for optimal task allocation



(b) Paths for optimal task allocation, computing paths with RRT



(c) Paths computed using parallel single-item auctions



(d) Paths computed using our algorithm

Figure 4: Layout in the simplified building experiments, comparing the optimal paths with the paths produced by parallel single-item auctions and by our algorithm.

length. The parallel single item auction results in assigning all the tasks in the upper half-plane to robot R_0, and those in the lower half plane to robot R_1, regardless of the order in which the tasks are auctioned. This increases the path length, as shown in Table 2. The performance of our algorithm is close to the optimal performance, and on par with starting with an optimal allocation.

Assuming that the underlying distributions are normal, we conducted unpaired two-tailed Welch's t-tests to compare the path lengths produced by our algorithm with the ones obtained when starting with an optimal allocation and the ones using parallel single-item auctions. There is a significant difference in the results for parallel single-item auctions vs. both the initial optimal allocation with RRTs ($t(37.66) = 26.458, p = 6.24 * 10^{-26}$) and our algorithm ($t(22.67) = 11.83, p = 3.54 * 10^{-11}$). This supports the hypothesis that parallel single-item auctions produce paths whose length is significantly different from optimal and from what our algorithm computes. The comparison between our algorithm and the initial optimal allocation with RRTs showed that the difference was not significant ($t(23.43) =$

Table 2: Length of minimum, maximum, and average path traveled in the experiments in the simplified building scenario. Path lengths are measured in meters. The environment is 16×16 meters. Results obtained over 20 runs. Results shown for optimal allocation, optimal initial allocation using RRTs, parallel single item auction, and our algorithm

Algorithm	Min path (m)	Max path (m)	Average path	
			Mean (m)	σ
Optimal using RRT	34.00	39.73	36.90	1.69
Parallel single-item	48.10	53.77	50.45	1.54
Our algorithm	33.07	50.98	36.78	4.93
Optimal path	31.64			

0.103, $p = 0.919$), supporting the observation that our algorithm performs well compared to the optimal allocation.

6.2. Lab scenario

For the real robot experiments, the robots were given a map of the lab which did not include chairs but included table positions, and were given a description of the team, including the wireless identifiers of the other robots. The robots started at different locations, and were given their own approximate position in the map. The tasks were scattered randomly in the lab and were initially divided equally between the two robots.

To ensure all tasks were done, when a robot had completed all its assigned tasks, it would wait a fixed amount of time (usually the amount of time the other robot had provided as its lowest bid) waiting for another robot to start a new auction. If any task in the system task list maintained by the robot was still incomplete and no auction had been started, the robot would start a new auction for the incomplete tasks.

The two experimental setups in the lab are illustrated in Figure 5 and Figure 6. The figures also show the RRTs formed by each robot in one of the runs.

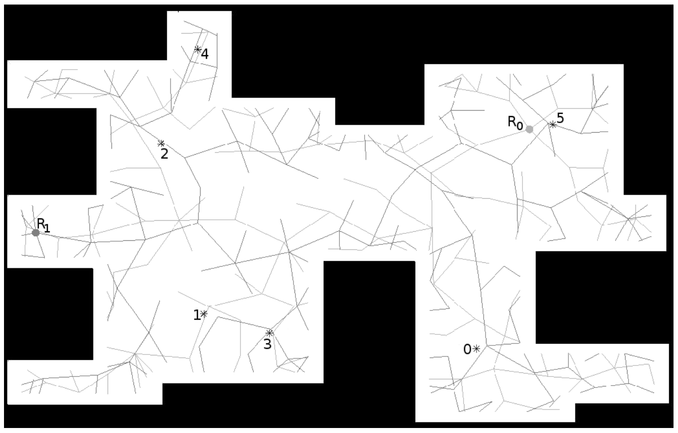


Figure 5: Experiment I map: robots are circles and tasks are asterisks. The RRTs for run 4 are shown.

In Experiment I there were six tasks scattered randomly in such a way that an optimal task allocation would result in an uneven distribution of the tasks between the robots. In Experiment II there were eight tasks distributed initially such that the majority of the tasks given to robot R_0 were closer to robot R_1 and vice versa. This was done to examine if the robots exchanged tasks successfully and completed them correctly.

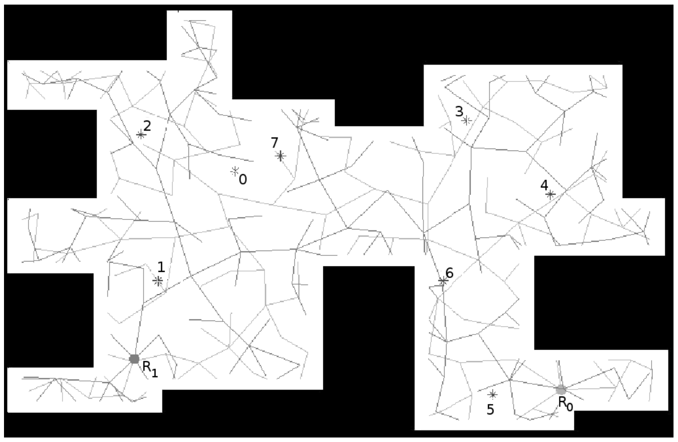


Figure 6: Experiment II map: robots are circles and tasks are asterisks. The RRTs for run 3 are shown.

We performed 5 runs of each experiment type individually, both in simulation and with the real robots.

The performance of the real robots in experiment II is shown in Figure 7. We can notice that the allocation of tasks is not the same in the different runs.For instance, in run number 4 of Experiment I, shown in Figure 5, task 0 was auctioned first but due to the way the RRT curved, the estimated cost for task 5 by robot R_0 was very high (it added the cost of going to and returning from task 0 to its cost estimate). Robot R_1 initially won task 2 because it had a lower cost estimate, but robot R_0 won it back after it completed task 0.

The task completion times for the lab scenario experiments are summarized in Table 3 and Table 4. In each case, the robots completed the assigned tasks within 2 minutes, staying well within the 10 minute time limit provided.

Table 3: Task completion times (in seconds) for Experiment I. Results shown are averaged over 5 runs.

Task ID	Assigned Robot		Real Robots		Simulation	
	Initial	Final	Mean (s)	σ	Mean (s)	σ
0	0	0	33.478	12.78	13.796	0.75
1	0	1	35.443	10.82	14.180	2.67
2	0	1	35.018	5.12	11.828	2.21
3	1	1	21.707	3.56	18.755	6.06
4	1	1	28.041	9.48	7.135	0.62
5	1	0	17.872	12.28	22.910	2.27
Total			121.618	16.53	52.955	7.01

In run number 3 in Experiment II (Figure 6), robot R_0 initially got stuck trying to get to task 6, and then completed the remaining tasks, but was much slower than usual in completing

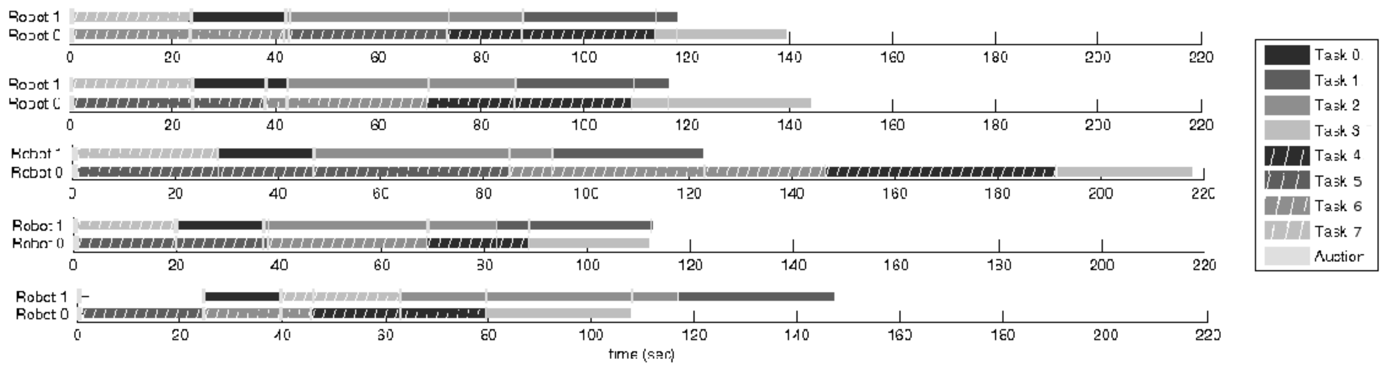


Figure 7: Experiment II real-robot timeline. Runs 1 through 5 (top to bottom)

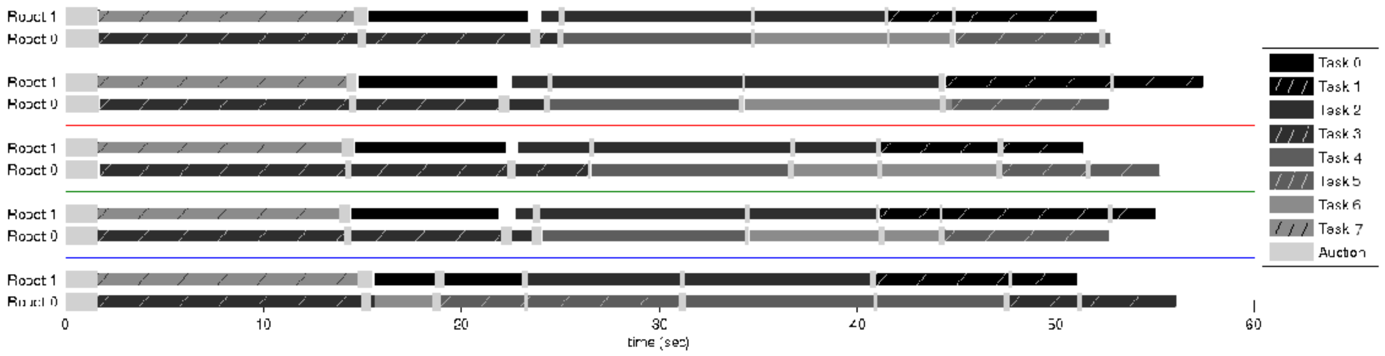


Figure 8: Experiment II simulation timeline. Runs 1 through 5 (top to bottom)

Table 4: Task completion times (in seconds) for Experiment II. Results shown are averaged over 5 runs.

Task	Assigned Robot		Real Robots		Simulation	
ID	Initial	Final	Mean (s)	σ	Mean (s)	σ
0	0	1	16.771	1.51	7.495	0.39
1	0	1	29.678	0.47	11.528	1.79
2	0	0	46.727	3.90	18.478	1.75
3	0	0	27.470	4.31	29.268	14.08
4	1	1	35.404	9.76	11.004	2.79
5	1	0	42.060	23.96	8.773	1.81
6	1	0	36.862	15.44	8.593	3.18
7	1	1	22.719	3.39	12.610	0.40
Total			151.185	39.04	53.642	1.72

the first two tasks, probably because of low battery.

The simulation experiments in comparison did not show robots getting stuck as often. The timeline for simulation experiment II is shown in Figure 8. A significant difference was a long initial auction time in simulation as compared to the real robots. This was likely caused by the fact that the computers used in the simulation shared a network and hence took longer to initially establish connections than the robots which had a dedicated network. This resulted in initial auction times on the order of 1.6 seconds in the first auction, dropping to 0.3 seconds subsequently. While the real robots also had a longer initial auction, such a large drop was not seen in the auction times.

Task completion times in simulation were significantly shorter than the corresponding times in the real robot experiments, as shown in Table 3 and Table 4.

Table 5: Auction times (in seconds) for Experiments I and II. Results shown are averaged over 5 runs.

Experiment	Real Robots		Simulation	
	Mean (s)	σ	Mean (s)	σ
I (6 tasks)	0.4052	0.1861	0.5527	0.5797
II (8 tasks)	0.4322	0.2412	0.4865	0.4938

The auctions took a very small percentage of the total time (as shown by the light grey bands in Figures 7 and 8, and summarized in Table 5), and caused small delays between one task and the next. This accounted for less than 1% of the time spent in performing the tasks. Communication time was also a very small fraction of the time taken to complete the tasks (on average, communications took up less than 1% of the work-time).

We can summarize the comparison between simulation and real robots as follows:

- Algorithm performance: The task allocation found in simulation was identical to that found in the real robot exper-

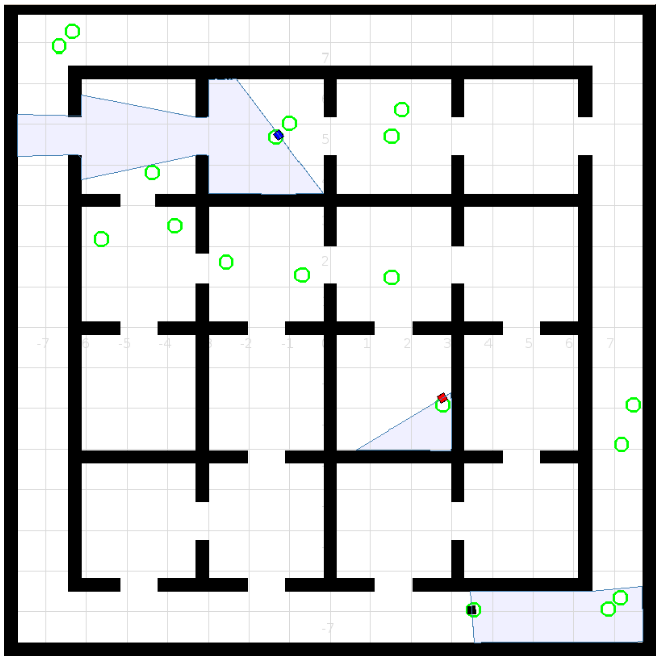


Figure 9: Stage image of the building scenario used in Experiment III.

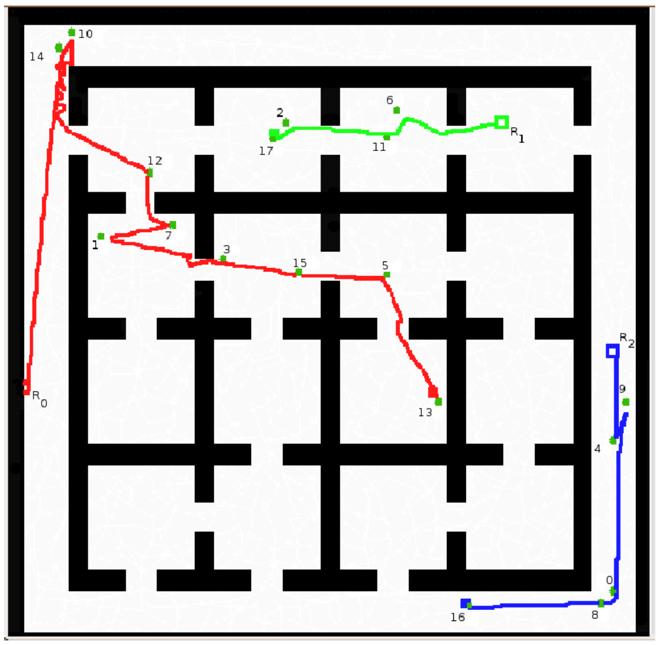


Figure 10: Simulation experiment III: An example showing the paths followed by robots R_0, R_1 and R_2.

iments, thus the simulation results were acceptable as predictors of the real robot performance. However, the impact of the time taken to perform the auctions was significantly less with the real robots compared to simulation, since execution times were much shorter in simulation.

- Time: the simulated robots moved faster than the real robots, despite the fact that we tried to find an equivalent velocity setting; thus, the auctions took a more significant portion of simulation time than they did in the real robot experiments. This speed difference also required modifications to the range parameter settings to get equivalent settings for the real robots as compared to simulation.
- Robot performance: The simulation was much more optimistic about the ability of the robots to detect obstacles and recover from errors; in the real robots, there was a tendency to get stuck that was not seen as frequently in simulation.

In conclusion, the simulation experiments were good indicators of real world performance, though some problems faced by actual robots were not perfectly mirrored in simulation.

6.3. Building scenario

We have also evaluated our auction algorithm in the environment described in [19], with 18 tasks and three robots (Figure 9). This environment is more complex than the lab environment, because there are numerous rooms and doors connecting them, so the navigation is harder. The major reason for choosing this environment is to enable comparison of results produced by different algorithms in the same environment. We used two different experimental setups. In Experiment III we used the same layout as the one used in [19]. In Experiment IV, we added moving obstacles in four locations (in the corridors and in front of doors) that hinder robot movement. We performed 10 runs for each of these experiments.

The paths followed by the robots in one of the runs for Experiment III are shown in Figure 10. The path followed by the robot on the left shows squiggly lines where the RRT was following the wall too closely. The obstacle avoidance routines would force the robot away from the wall, but the path to be followed would bring it back close to the wall. This kind of movement was happening often due to the tendency of RRT nodes to be generated close to walls when in an environment with many rooms. However, this did not cause a significant negative impact on the motion of the robot, when overall performance is considered.

The experiments show that the robots were able to successfully complete the tasks scattered in the environment, generating paths comparable to those shown in [19]. However, a direct comparison with [19] is not possible due to differences in scale, number of tasks, and positions.

In Figure 11 we show the environment used for Experiment IV. There are four obstacles, shown as small rectangles, that move across the corridor or in front of a door. The paths followed by the robots in one of the runs for Experiment IV are shown in Figure 12.

The experiments with moving obstacles showed only small differences from the ones without obstacles, as can be seen in Table 6. In the runs with obstacles the robots successfully coped with moving obstacles, showing on average only a 5% increase in path length. Similarly completion time averaged 6 min and 47 sec without obstacles, and showed an increase of approximatively 10% (to 7 min and 30 sec) in the case of obstacles. The variance in average distance traveled was greater in the runs

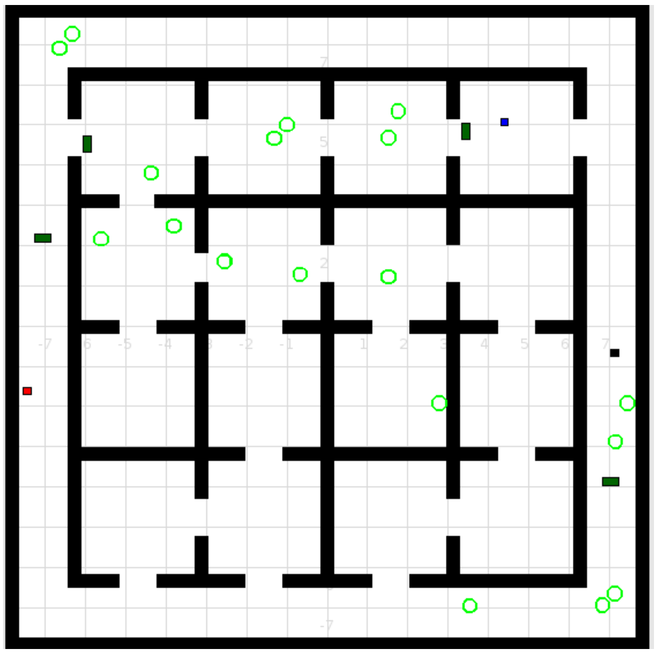


Figure 11: Stage image of the building scenario with obstacles used in Experiment IV. The obstacles move along their longer axis.

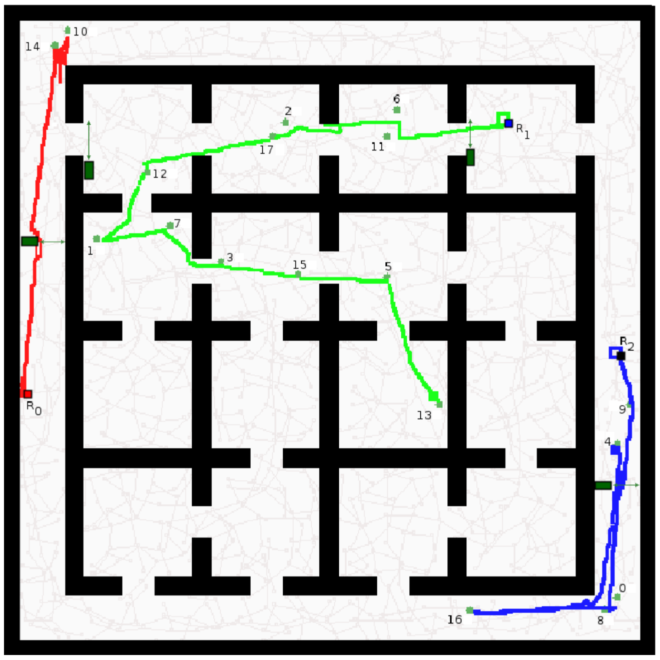


Figure 12: Simulation Experiment IV: an example showing the paths followed by the robots when obstacles are present.

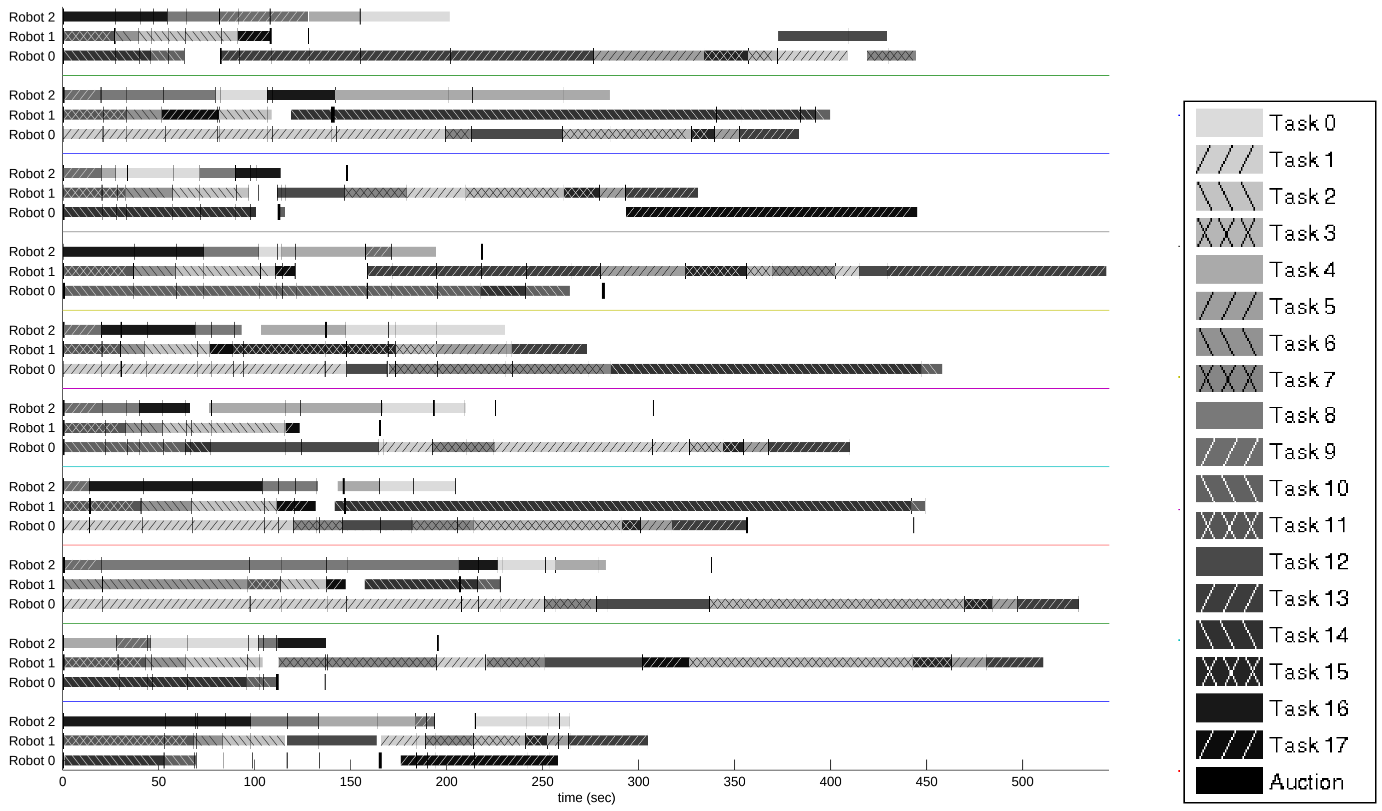


Figure 13: Experiment IV timeline in the environment shown in [19] - with obstacles.

with obstacles, as expected. The robots dealt with obstacles by auctioning tasks again, and trying to access blocked areas repeatedly until the tasks in those areas were completed.

The timeline in Figure 13 shows the length (by task) of the path followed by each robot in each run in Experiment IV. Short gaps indicate intervals where the robot was attempting a task that was completed by a different robot later (it counts as part of the distance traveled by the robot, but is not productive in terms

Table 6: Average path and longest path traveled in Experiments III (no obstacles) and IV (with obstacles). Path lengths are measured in meters. The size of the environment is 16×16 meters.

Experiment	Average path		Average longest path	
	Mean (m)	σ	Mean (m)	σ
III (no obstacles)	21.14	1.91	31.53	7.12
IV (obstacles)	22.31	2.22	33.62	5.77

of task completion). The large gaps are intervals where a robot had completed all its tasks, and took on another robot's tasks if the other robot was getting delayed too long. This is done as a means to ensure that as many tasks as possible are completed within the time limit (the overarching objective), thus allowing for some inefficiency in favor of completeness.

One difference we noted with previous experiments was that the robots had a tendency to follow a different order of task completion in each run. This is likely due to the environment and the RRT paths. The re-ordering did not appear to affect performance in terms of average path traveled by the three robots, however it did affect the length of the longest path traveled, as shown in the difference between runs 4 and 10 in Figure 13.

Variations in the order in which tasks were accomplished was caused primarily by the RRTs which tend to bias distances according to the manner in which the RRT tree was formed. In an environment like this, with many ways to access the same room, different experimental runs would often find different non-overlapping routes to the tasks. Despite this effect, the distance traveled did not show too great a variation between runs.

7. Conclusions and Future Work

We have presented an algorithm based on auctions for allocation of tasks to robots, which is robust to robot failure and environmental uncertainty. We have analyzed the algorithm's complexity and compared it with other algorithms in current use.

The experiments with real robots showed performance similar to those done in simulation, even if the real robots were slower than the simulated ones and more prone to problems. The experiments showed that the task allocations found did not suffer significantly from the change in speed in the robots. As a side effect, the ratio of time for the auctions to the time to execute the tasks was significantly smaller in the experiments done with real robots.

The robots proved adaptable, tasks were exchanged during execution, and the final task assignment was close to optimal. The comparison of performance between simulation and real robots showed that simulation results may be relied on.

Future work will include stressing the algorithm even more with multiple failures of the robots and with repeated communication failures. Specifically we want to address the case of communication failures before a robot had time to share its tasks with the other robots. We are also extending our previous work on auctions for allocation of tasks with precedence constraints and task duration [4] to work with robots, as a way of providing a more static allocation of tasks for situations where tasks have interdependencies.

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